

Nuclear Energy, the Energy Balance

Chapter 5 Technical/Mathematical Summary of the Formulas, second revision

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Presentation of the data

The great variety in the types of data used to calculate the energy costs of nuclear energy makes it difficult to give a uniform picture. The mix of electrical energy and thermal energy in the different processes makes it hard to give a coherent picture. We have chosen to give the total energy and the *ratio*, R , between the thermal component and the electrical component. The energy unit is always *petajoules* (PJ) and the weight unit (when applicable) is always *megagram* (Mg). In the equations the mass (e.g., the mass of enriched uranium loaded into the reactor) is always given by M . We have calculated the energy costs of thirteen steps, or items, in the fuel cycle (fourteen steps, if mining and milling are counted separately).

1. Mining and milling

In Chapter 2, on the basis of references of the industry, the energy cost of mining and milling of soft ores is calculated to be:

$J_{mm} = J_e + J_{th} = 2.33$ GJ/Mg ore with $R=J_{th}/J_e = 7.5$, and for hard ores:

$J_{mm} = J_e + J_{th} = 5.55$ GJ/Mg ore with $R=J_{th}/J_e = 1.6$.

Using these values the energy expenditure of the mining and milling processes are calculated by means of Eq.2.1 and 2.2. These expressions are explained in detail in Chapter 2.

$$J = j(c, G) = \frac{c}{Y \bullet G} \quad \text{GJ/kg U} \quad (\text{Eq. 2.1})$$

$$Y = 0.980 - 0.0723 \bullet (\log G)^2 \quad (\text{Eq. 2.2})$$

with:

$$c = J_{mm}/8.48 \quad \text{GJ/kg U}$$

Y = Yield of the mining and milling processes.

G =ore grade (mass-% U_3O_8)

The value of c in two different units, for hard and for soft ores, is given in Table 4.

Table 4

| ore type | c (GJ/kg U) | c (PJ/Mg U) | $R= J_{th}/J_e$ |
|----------|---------------|---------------|-----------------|
| soft | 0.275 | 0.000275 | 7.5 |
| hard | 0.654 | 0.000654 | 1.6 |

The total energy expenditure for milling and mining of one reactor load of mass M is given by:

$$M \bullet j(c, G) \bullet f(x_p, x_f, x_t)$$

See the paragraph on enrichment, below, for the definition of $f(x_p, x_f, x_t)$.

2. Conversion of U_3O_8 into UF_6

The specific conversion energy expenditure is given by:

$$J_2 = 0.00148 \text{ PJ/MgHM} ; R=27.0 \quad [\text{Erda-76-1}] \quad (\text{Eq.3})$$

Loss 0.5% [NRC,1996]

The total conversion energy expenditure for one reactor load of mass M is:

$$J_2 \bullet M \bullet f(x_p, x_f, x_t)$$

Enrichment

The separative work S (number of separative work units SWU) can be calculated from DOE/EIA, 1997:

$$\frac{F}{P} = f(x_p, x_f, x_t) = \left(\frac{x_p - x_t}{x_f - x_t} \right) \quad (\text{Eq. 4.1})$$

$$S(P, x_p, x_f, x_t) = P \cdot [V(x_p) - f(x_p, x_f, x_t) \cdot V(x_f) + (f(x_p, x_f, x_t) - 1) \cdot V(x_t)] \quad (\text{Eq. 4.2})$$

$$V(x) = (2x - 1) \cdot \ln\left(\frac{x}{1-x}\right) \quad (\text{Eq. 4.3})$$

| | |
|------------------------------|----------------------------|
| F = feed mass | Mg U |
| P = product mass | Mg U |
| S = separative work | SWU =separative work units |
| x_f = feed assay = 0.0071 | fraction ^{235}U |
| x_p = product assay | fraction ^{235}U |
| x_t = tails assay = 0.0020 | fraction ^{235}U |

The resulting values of S for several typical cases are:

$$\text{for } x_p = 2.6\% \text{ } ^{235}\text{U} \quad S = 3.45 \text{ Mg SWU/Mg U}$$

$$\text{for } x_p = 3.3\% \text{ } ^{235}\text{U} \quad S = 4.98 \text{ Mg SWU/Mg U}$$

$$\text{for } x_p = 4.2\% \text{ } ^{235}\text{U} \quad S = 7.01 \text{ Mg SWU/Mg U}$$

We have calculated the energy costs of enrichment based on the assumption of a 70/30 world-wide ratio of centrifuge to gas-diffusion enrichment. As explained in Chapter 2 we have also introduced the unit, Mg SWU, equal to 1000 SWU.

3. Enrichment

Specific energy expenditure of gas-diffusion enrichment:

$$J_3 (\text{gd}) = \mathbf{0.011 \text{ PJ/Mg SWU}; \mathbf{R=0.083} \quad [\text{Erda-76-1}] \quad (\text{Eq. 5.1})$$

$$\text{Loss 0.5\%} \quad [\text{NRC, 1996}]$$

Specific energy expenditure of centrifuge enrichment:

$$J_3 (\text{uc}) = \mathbf{0.00310 \text{ PJ/Mg SWU}; \mathbf{R=2.72} \quad (\text{Eq. 5.2})$$

Using above specified mix we get:

$$J_3 (\text{mix}) = \mathbf{0.0055 \text{ PJ/Mg SWU}; \mathbf{R= 0.51} \quad (\text{Eq. 5.4})$$

The energy expenditure for enrichment of one reactor load is thus:

$$J_3 \cdot S(M, x_p, x_f, x_t)$$

4. Fuel element fabrication:

Specific energy expenditure

$$J_4 = \mathbf{0.00378 \text{ PJ/Mg U}; \mathbf{R=2.50} \quad [\text{Erda -76-1}] \quad (\text{Eq. 6})$$

$$\text{Loss 1\%} \quad [\text{NRC, 1996}]$$

The energy cost for the fabrication of the fuel for one reactor load, mass M is:

$$J_4 \cdot M$$

5. Returning the mining area to "green field" conditions:

The mass of mill tailings is given by:

$$m_t(M, G, x_p, x_f, x_t) = 1.18 \cdot M \cdot f(x_p, x_f, x_t) \cdot \left(\frac{100}{Y(G) \cdot G} - 1 \right) \quad (\text{Eq. 11})$$

The factor 1.18 comes from converting Mg U to Mg U_3O_8 : 1.18 Mg U_3O_8 contains 1 Mg U. The factor 100 comes from following the convention of specifying ore fractions in percent.

As explained in Chapter 3 it is essential that the mill-tailings be sequestered in such a way that the environment is not endangered. We estimate the energy cost of doing this to be:

$J_5 = 4.5 \times 10^{-6}$ PJ/Mg, $R = 8.0$; (the mass (Mg) refers to the mill-tailings mass, not the uranium).

The energy cost associated with restoring the mining and milling area for one reactor load to green field conditions for is thus:

$$J_5 \bullet m_t(M, G, x_p, x_f, x_t)$$

Spent fuel elements

As is well known, this is a most intractable question. The formidable safety problems involved are not the subject of this study however. We have estimated the costs, based on mining experience, and our estimate turned out to be very close to the figures from the Swedish SKB-3 concept. It is still unknown if this is a viable solution, but one may assume that if it is, it will be a safe solution.

6. Interim storage

As discussed in Chapter 4, a long storage period is needed to allow the radioactivity to decay to a level where the fuel elements can be handled (albeit under remote control behind heavy shielding walls). The cost of this phase is estimated to be **$J_6 = 0.0095$ PJ/MgU, with $R = 5$.**

The energy expenditure for the interim storage of one reactor load, mass M is:

$$J_6 \bullet M$$

7. Conditioning

The spent fuel elements are placed in V5 containers and sealed off, under remote control. This process is described in more detail in Chapter 4. The energy expenditure is **$J_7 = 0.002$ PJ/Mg with $R=0.11^3$.**

The energy cost of conditioning the fuel from one reactor load, mass M is:

$$J_7 \bullet M$$

8. Disposal

The extremely difficult process of disposal is described in Chapter 4. The energy expenditure is estimated at **$J_8 = 0.01$ PJ/Mg with $R=8^1$.**

The energy cost of a reactor load, mass M is:

$$J_8 \bullet M$$

9. Operation, maintenance and refurbishing (omr)¹

These costs are simply proportional to time, on the average. They are estimated (see Chapter 3) at:

$J_9 = 2.0$ PJ/300 days, with $R=10.6$

This item simply costs J_9 per reload period.

Operational wastes

10. Conditioning

We have taken the figures from the industry based on average amounts (see Chapter 4). The sequestration of the waste products of conversion, and fuel fabrication are included in the total, along with the operational wastes of the reactor itself (this does not include the fuel elements which are a special case treated separately above; the enrichment wastes are also treated separately, below). Although it is an average value, it is probably a good approximation. The conditioning (packaging) of these wastes is described in detail in Chapter 4. This amounts to filling 1895 V2 containers per GW(e).a at a total energy cost of 0.535 PJ. The total energy expenditure for a reload period of 300 days is therefore $0.535 \times 300/365$ PJ or:

$J_{10} = 0.440$ PJ, with $R = 4.8$. Note that this is treated as a constant per unit of time (300 days).

The cost is here again just J_{10} per reload period.

11. Disposal

The costs of the disposal of these wastes is much less than for the highly dangerous spent fuel elements. It amounts to (Chapter 4) 45.5 GJ/m³ waste. Since the V2 container has a volume of 1 m³ the total cost is just equal to 45.5 GJ per container. In the reload period of 300 days this comes to $1895 \times 300/365$ or 1558 containers. The total disposal costs are then:

$J_{11} = 0.0708$ PJ, with $R = 8.0$.

Again the costs are J_{11} per reload period.

12. Depleted uranium

The amount of depleted uranium is calculated from Eq. 8, which can be derived from Eq.4.1, Chapter 2.

$$w(P, x_p, x_f, x_t) = F - P = P \left(\frac{x_p - x_f}{x_f - x_t} \right) \quad (\text{Eq. 8})$$

This should be safely packed (in V2 containers), after having been reconverted from the volatile form UF₆ to U₃O₈, and finally disposed of in a geological repository. The energy expenditures required are calculated as follows:

| | | |
|---------------------------------------|-----------------|-----------------|
| Reconversion (same as conversion) | 1430 GJ(th)/MgU | + 53 GJ(e) /MgU |
| Conditioning (see Chapter 4) | 166 GJ(th)/MgU | + 35 GJ(e)/MgU |
| Disposal (see Chapter 4) ² | 28 GJ(th)/MgU | + 4 GJ(e)/MgU |
| Total (rounding) | 1620 GJ(th)/MgU | + 90(e)Mg/U |

giving a total energy requirement for the safe disposal of the depleted uranium “left behind” by the various processes described of **$J_{12} = 0.0017 \text{ PJ/MgU with R= 18}$**

The energy costs for a reactor load are thus:

$$J_{12} \bullet w(M, x_p, x_f, x_t)$$

13. Enrichment waste

The waste from the enrichment process is (weakly) radioactive, and must also be safely disposed of.

The energy costs of this disposal were calculated in Chapter 4. They are:

| | | |
|-----------------|--|---|
| Conditioning: | $6.5 \times 10^{-6} \text{ PJ(e)/Mg SWU}$ | + $66.8 \times 10^{-6} \text{ PJ(th)/Mg SWU}$ |
| Disposal | $1.3 \times 10^{-6} \text{ PJ(e)/Mg SWU}$ | + $10.5 \times 10^{-6} \text{ PJ(th)/Mg SWU}$ |
| With a total of | $7.8 \times 10^{-6} \text{ PJ(th)/Mg SWU}$ | + $77.3 \times 10^{-6} \text{ PJ(th)/Mg SWU}$, |

or

$$J_{13} = 85.1 \times 10^{-6} \text{ PJ/Mg SWU with R = 9.9.}$$

The energy costs are thus:

$$J_{13} \bullet S(M, x_p, x_f, x_t)$$

Total net energy production from nuclear power

In Chapter 2 it was shown that the total known reserves of uranium would only be capable of supplying the present day world-wide electrical energy consumption for less than a decade. As we have remarked earlier, this amount is so small one must ask oneself why it is that nuclear power was ever considered as holding promise of very large amounts of energy. The answer may be that on the one hand it was imagined that the fast-neutron breeder reactor would quickly be developed, and on the other hand and would work as designed, and on the other there was no realization of the immense energy costs of operating nuclear reactors which we have calculated in this document. In addition it was not realized for many years how dangerous radioactive waste was, and that the problem of its disposal would be so intractable.

Capitalization of the debts

In Chapter 3 the total energy costs of construction and dismantling were estimated at 240 PJ. In one option we assume this debt will be paid in the years and decades to come. In the other option we assume that society (the nuclear-energy industry in particular) will simply abandon old reactors, reducing the debt to 80 PJ. We have capitalized the debt at the moment that the reactor begins to work. Those accustomed to economic calculations (using money as the descriptor of value) are aware that the date at which one assumes a debt is of crucial importance, because of the interest which must be paid and the ensuing discounting of value. This problem does not exist if energy units are used, since energy is a real, conserved quantity, whereas money has no value beyond that which people (arbitrarily) assign to it.

Equations of the graphs

In the description of the calculations detailed below, it is implicit that the type of ore is specified.

Net-energy production

The graphs shown in Figures 8a, 8b, 9a and 8b, in Chap 2, connect three points with two straight lines.

The first point, on the y-axis, is simply the debt, i.e, in figures 8, 240 PJ, in figures 9, 80 PJ

The second point has an x-value of 300 days, or 0.822 full-load years.

The y-increment of this point (from the first point) is the energy production in 300 days, or 25.92 PJ minus the sum of the thirteen energy costs (where mining and milling are lumped together) listed in the table at the end of the Introduction, and given quantitatively in the previous section of this chapter, where the initial load mass and fuel assays are substituted in the formulas to evaluate the costs.

The third point has an x-value of $0.822 \times (\text{number of reloads} + 1)$.

The y-increment with respect to the second point is the number of reloads, plus one, multiplied by the energy production minus the thirteen energy costs. The reload mass and fuel assay is substituted in the equations.

Relative CO₂ emission

The graphs shown in Figure 6 of Chapter 1 give the ratio of the CO₂ emission of a nuclear system to that of a gas-burning system with the same **net** electrical output, as a function of the ore grade. The calculation of these curves is described in detail in Chapter 1.
